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Direction ratios, direction cosines and lines in space

Formulas and units, JEE Main. Status: Reviewed on 2026-11-20.

Result	Formula	Units or note
Direction cosines	l = cos alpha, m = cos beta, n = cos gamma; l^2 + m^2 + n^2 = 1 Valid when: direction cosines of any directed line in space	dimensionless
Ratios to cosines	l = a/r, m = b/r, n = c/r, r = sqrt(a^2 + b^2 + c^2) Valid when: a, b, c not all zero, direction ratios of the line	dimensionless
Ratios of the line PQ	(x2 - x1, y2 - y1, z2 - z1) Valid when: any two distinct points P and Q	length
Line through A with ratios a, b, c	(x - x1)/a = (y - y1)/b = (z - z1)/c Valid when: a, b, c not all zero; a zero ratio means that coordinate is constant	n/a (coordinates)
Angle between two lines	cos(theta) = |a1 a2 + b1 b2 + c1 c2|/(r1 r2) Valid when: a, b, c are direction ratios; gives the acute angle	dimensionless
Perpendicular lines	a1 a2 + b1 b2 + c1 c2 = 0 Valid when: direction ratios of two lines in space	dimensionless
Parallel lines	a1/a2 = b1/b2 = c1/c2 Valid when: ratios proportional; a zero ratio matches a zero ratio	dimensionless
Sine identity for the axes	sin^2 alpha + sin^2 beta + sin^2 gamma = 2 Valid when: direction angles alpha, beta, gamma of one line with the axes	dimensionless
Foot of perpendicular parameter	t = ((P - A) . d)/(d . d) Valid when: A on the line, d its direction; F is the foot from P	dimensionless

Full lesson with worked examples:
https://www.engineeringrise.in/learn/learn/jee-main/mathematics/three-dimensional-geometry/direction-ratios-direction-cosines-and-

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